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**SYNCHROTRON RADIATION CONSIDERATIONS IN THE  
DENSE PLASMA FOCUS ( DPF ) MAGNETOPLASMA-  
DYNAMIC ( MPD ) THRUSTER**

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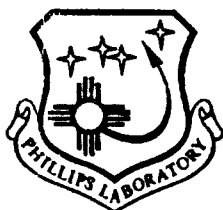
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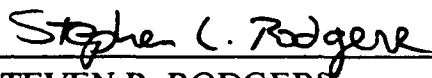
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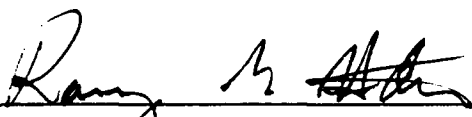
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13. ABSTRACT The Dense Plasma Focus (DPF) computer model has been modularized and updated. The code now assumes an inherent value $\beta = 1$ upon which further calculations are based. The DPF model is using deuterium (D) and Helium-3 ( $^3\text{He}$ ) as the fuels and hydrogen as the propellant. D-D side reactions have been included. $I_{sp}$ values in the range of 900-1000 have been obtained for operating temperatures in the tens of keV's regime, as well thrust-to-weight ratios on the order of $10^{-4}$ . The majority of the work has been devoted to the synchrotron radiation emissions from the DPF. It has been shown that for $\beta = 1$ , no synchrotron radiation will be emitted. If the $\beta$ value is $< 1$ , then reabsorption of the synchrotron radiation will become a concern for plasma operating temperatures exceeding $\sim 62$ keV. Calculations have been made assuming a plasma pinch mass density of $2.2 \times 10^{-4} \text{ kg} \cdot \text{m}^{-3}$ . The characteristic synchrotron radiation absorption length, $\lambda_0$ , for a $\beta = 1$ case has been found to be less than the radius of the plasma pinch.				
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## TABLE OF CONTENTS

Section	Page
INTRODUCTION AND REPORT SUMMARY	1
GLOSSARY OF SYMBOLS	2
ELECTROMAGNETIC RADIATION LOSSES IN THE DPF	4
ROLE OF ELECTRIC AND MAGNETIC FIELDS	4
BREMSSTRAHLUNG	4
SYNCHROTRON RADIATION	7
EFFECT OF PLASMA BETA ON SYNCHROTRON RADIATION	
EMISSION	11
ABSORPTION OF SYNCHROTRON RADIATION	14
DENSE PLASMA FOCUS COMPUTER MODEL	19
THE DENSE PLASMA FOCUS: A FUSION PROPULSION	
SCENARIO	19
A DENSE PLASMA FOCUS MODEL (REVISED DENSE	
PLASMA FOCUS CODE)	20
CONCLUSIONS AND RECOMMENDATIONS	25
REFERENCES	26
ACKNOWLEDGEMENTS	27
APPENDIX A	28
APPENDIX B	44

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## LIST OF FIGURES

Figure	Caption	Page
1	Dense Plasma Focus Overview	5
2	Cylindrical Coordinate System	9
3	$P_{\text{brems}}$ [MW], $P_{\text{sync}}/P_{\text{brems}}$ vs. $K$ [keV] for $n_e = 3.534\text{E}+26 \text{ m}^{-3}$	15
4	Power [MW] vs. $K$ [keV] for $n_e = 3.534\text{E}+26 \text{ m}^{-3}$	16
5	Plasma Sheath ("Snowplow") Rundown	22

## INTRODUCTION AND REPORT SUMMARY

As mankind sets its sights on the next threshold of space exploration (manned expeditions to neighboring planets), propulsion concepts need to be developed to meet the requirements of such missions. Chemical propulsion systems can meet the criteria needed for nearby planetary trips, but nuclear fusion offers better performance in specific impulse and trip duration [Ref. 1]. One fusion propulsion concept which the Air Force is studying uses the Dense Plasma Focus (DPF).

The DPF propulsion concept was first studied in-house by the Air Force at the Phillips Laboratory by C. Leakeas [Ref. 1]. His report modeled a DPF propulsion system in its entirety. The work herein serves to streamline and update the previous computer model and also to determine the effects of electromagnetic (EM) radiation losses on the DPF performance. After beginning with a short discussion of the cause of such losses, the two major forms of EM radiation losses in fusion are explained in detail, those being bremsstrahlung and synchrotron radiation.

A primary goal of this report is to investigate the effects of the synchrotron radiation losses on the DPF concept. Special emphasis is placed on determining the synchrotron radiation's emission in relation to the plasma pressure ratio parameter  $\beta$ . The quantity of synchrotron radiation in relation to bremsstrahlung is mentioned as well. Further study is done to find the degree to which emitted synchrotron radiation may be reabsorbed by the plasma.

After looking at the effects of the EM losses, the remainder of the report is devoted to describing the new DPF computer code. The ideas and assumptions used in the original program are given, along with the alterations and reasons for their presence in the new code. The effects on DPF rocket performance as a result of the new code are mentioned briefly. Conclusions of the current research and recommendations for future research in both the areas of DPF EM losses and DPF computer modeling are given in closing this report.

## GLOSSARY OF SYMBOLS

<b>a</b>	= Acceleration [ $\text{m} \cdot \text{s}^{-2}$ ]
<b>a<sub>r</sub></b>	= Radial acceleration [ $\text{m} \cdot \text{s}^{-2}$ ]
<b>B</b>	= Magnetic flux density [T]
<b>B<sub>i</sub></b>	= Magnetic flux density within the plasma [T]
<b>B<sub>o</sub></b>	= Outer, or surface, magnetic flux density [T]
<b>β</b>	= Plasma pressure ratio parameter
<b>β<sub>e</sub></b>	= Electron pressure ratio parameter
<b>β<sub>i</sub></b>	= Ion pressure ratio parameter
<b>c</b>	= Speed of light in a vacuum ( $2.99792458 \text{ [m} \cdot \text{s}^{-1}\text{]}$ )
<b>c'</b>	= Numerical coefficient [ $\text{T}^{-2} \cdot \text{s}^{-1}$ ]
<b>c<sub>p,D</sub></b>	= Specific heat of deuterium [ $\text{J} \cdot \text{kg}^{-1} \cdot ^\circ\text{K}^{-1}$ ]
<b>γ</b>	= Relativistic mass correction factor
<b>D<sub>p</sub></b>	= Plasma depth [m]
<b>Δt</b>	= Length of time in one pulse [s]
<b>ΔT</b>	= Change in temperature of electrodes during one pulse [K]
<b>e</b>	= One electronic charge ( $1.6021892 \times 10^{-19} \text{ [C]}$ )
<b>E</b>	= Electric field strength [ $\text{V} \cdot \text{m}^{-1}$ ]
<b>E<sub>e0</sub></b>	= Electron rest mass energy (511.0 [keV])
<b>η</b>	= Bremsstrahlung classical treatment parameter [Ref. 4]
<b>η<sub>e</sub></b>	= Electrical resistivity [ $\Omega \cdot \text{m}$ ]
<b>η<sub>tot</sub></b>	= Total particle collection efficiency in plasma pinch
<b>f(p)</b>	= Relativistic Maxwellian momentum distribution function
<b>F</b>	= Force [N]
<b>g</b>	= Gaunt relativistic correction factor for Bremsstrahlung
<b>h</b>	= Planck's constant ( $6.626176 \times 10^{-34} \text{ [J} \cdot \text{s]}$ )
<b>I<sub>max</sub></b>	= Electrical current needed to create required magnetic flux density in the DPF [A]
<b>k</b>	= Boltzmann's constant ( $1.380662 \times 10^{-23} \text{ [J} \cdot \text{K}^{-1}\text{]}$ )
<b>K</b>	= Plasma temperature in [keV], (see also T [ $^\circ\text{K}$ ])
<b>K<sub>e</sub></b>	= Electron temperature in [keV]
<b>K<sub>i</sub></b>	= Initial kinetic energy in [keV]
<b>K<sub>f</sub></b>	= Final kinetic energy in [keV]
<b>K<sub>2</sub>( )</b>	= Modified Bessel function of 2nd order
<b>K<sub>3</sub>( )</b>	= Modified Bessel function of 3rd order
<b>λ<sub>0</sub></b>	= Characteristic synchrotron radiation absorption length [m]
<b>m<sub>av</sub></b>	= Average mass of a particle in the pinch [kg]
<b>m<sub>D</sub></b>	= Mass flow of deuterium [ $\text{kg} \cdot \text{s}^{-1}$ ]
<b>m<sub>e</sub></b>	= Rest mass of electron ( $9.1095346 \times 10^{-31} \text{ [kg]}$ )
<b>μ<sub>0</sub></b>	= Permeability of free space ( $4\pi \times 10^{-7} \text{ [H} \cdot \text{m}^{-1}\text{]}$ )
<b>n<sub>beg</sub></b>	= Initial pinch ion density [ $\text{m}^{-3}$ ]
<b>n<sub>D</sub></b>	= Number density of deuterons [ $\text{m}^{-3}$ ]
<b>n<sub>e</sub></b>	= Electron number density [ $\text{m}^{-3}$ ]
<b>n<sub>i</sub></b>	= Species 'i' ion number density [ $\text{m}^{-3}$ ]
<b>n<sub>He</sub></b>	= Number density of <sup>3</sup> He ions [ $\text{m}^{-3}$ ]
<b>N<sub>e</sub></b>	= Number of electrons
<b>ν</b>	= Photon frequency [ $\text{s}^{-1}$ ]

$p$  = Linear momentum [ $\text{kg} \cdot \text{m} \cdot \text{s}^{-1}$ ]  
 $p_r$  = Linear momentum in radial direction [ $\text{kg} \cdot \text{m} \cdot \text{s}^{-1}$ ]  
 $p_\theta$  = Linear momentum in tangential direction [ $\text{kg} \cdot \text{m} \cdot \text{s}^{-1}$ ]  
 $p_z$  = Linear momentum in z direction [ $\text{kg} \cdot \text{m} \cdot \text{s}^{-1}$ ]  
 $P_B$  = Bremsstrahlung power loss density [ $\text{W} \cdot \text{m}^{-3}$ ]  
 $P_{\text{brems}}$  = Bremsstrahlung power [W]  
 $P_{\text{cyc}}$  = Cyclotron power [W]  
 $P_{\text{ohm}}$  = Ohmic heating power [W]  
 $P_r$  = Radial power [W]  
 $P_{\text{sync}}$  = Synchrotron power ( $= dW_e/dt$ ) [W]  
 $P_{\text{tot}}$  = Total radiated power ( $= dW/dt$ ) [W]  
 $P$  = Particle kinetic pressure [Pa]  
 $q$  = Total electronic charge on a particle [C]  
 $q_e$  = Electronic charge on an electron ( $= -e$ ) [C]  
 $r_{\text{gyr}}$  = Radius of gyration [m]  
 $r_p$  = Plasma radius [m]  
 $R$  = Effective reflectivity of emitted synchrotron radiation  
 $\rho_{\text{init}}$  = Initial mass density of fuel particles in DPF entrance [ $\text{kg} \cdot \text{m}^{-3}$ ]  
 $T$  = Plasma temperature in [ $^{\circ}\text{K}$ ], (see also  $K$  [keV])  
 $T_e$  = Electron temperature in [ $^{\circ}\text{K}$ ]  
 $\mathbf{v}$  = Velocity [ $\text{m} \cdot \text{s}^{-1}$ ]  
 $\mathbf{v}_\theta$  = Tangential velocity [ $\text{m} \cdot \text{s}^{-1}$ ]  
 $v_e$  = Speed of electron [ $\text{m} \cdot \text{s}^{-1}$ ]  
 $v_1$  = Speed of particle 1 [ $\text{m} \cdot \text{s}^{-1}$ ]  
 $V_{\text{init}}$  = Initial volume of rundown (interelectrode) chamber [ $\text{m}^3$ ]  
 $V_{\text{pin}}$  = Volume of plasma pinch [ $\text{m}^3$ ]  
 $Z$  = Average atomic number of the fusion species  
 $Z_i$  = Number of charges on ion species 'i'  
 $Z_1$  = Number of charges on particle 1  
 $Z_2$  = Number of charges on particle 2  
 $\omega$  = Angular frequency [ $\text{s}^{-1}$ ]  
 $\omega_c$  = Cyclotron angular frequency [ $\text{s}^{-1}$ ]  
 $\omega_{ce}$  = Electron cyclotron angular frequency [ $\text{s}^{-1}$ ]  
 $\omega_{pe}$  = Electron plasma angular frequency [ $\text{s}^{-1}$ ]



## **ELECTROMAGNETIC RADIATION LOSSES IN THE DPF**

### **ROLE OF ELECTRIC AND MAGNETIC FIELD LINES**

In this section, the fields and interactions in and around the DPF plasma pinch region are analyzed. Following the lead of earlier work, the pinch will be approximated as having a cylindrical shape. An illustration of the DPF and pinch may be found in Figure 1. The view is a cross section, showing the annular, cylindrical cathode with the rod-like anode in the center. The enclosed portion of the device extending from the end of the cathode to the nozzle entrance is the mixing chamber. In it, the remaining fusion reactants and products mix with the propellant, heating it. This results in a high temperature gas mixture, which is expanded through the nozzle to produce thrust. A dash-lined ellipse surrounds the area known as the pinch. When necessary, the pinch dimensions used in this report are radius = 0.0015 [m] and length = 0.0254 [m]. These are the values assumed by Leakeas [Ref. 1].

Charged particles exhibit changes in their motions and emit radiation due to additional kinetic energy imparted through accelerations. These accelerations arise from interactions with electromagnetic fields. Because plasmas are composed of ions and electrons, interactions involving them are the key concerns. The most prominent examples are the interaction of electrons with electric fields (bremsstrahlung) and the interaction of electrons with magnetic fields (synchrotron radiation). Both of these radiation types will be covered, with a special emphasis given to synchrotron radiation, defined in the **SYNCHROTRON RADIATION** section. Its dependence on the parameter  $\beta$  for emission is detailed in the **EFFECT OF PLASMA BETA ON SYNCHROTRON RADIATION** section. Its reabsorption following emission is discussed in the **ABSORPTION OF SYNCHROTRON RADIATION** section.

### **BREMSSTRAHLUNG**

Bremsstrahlung, German for "braking radiation" [Ref. 2], is the radiation emitted by a moving charged particle accelerated by collisions with other particles. Because a particle's acceleration is inversely proportional to its mass for a given force, the bremsstrahlung emitted by ions is much less than that of electrons. The electrons, however, are deflected (accelerated) much more by the ions present than by other electrons. Thus, in the field of fusion, bremsstrahlung is usually considered to emanate from the interaction of moving electrons with essentially stationary ions.

Moving electrons constantly come in contact with the electric fields arising from other charged particles (either ions or other electrons). Bremsstrahlung can be emitted through a process known

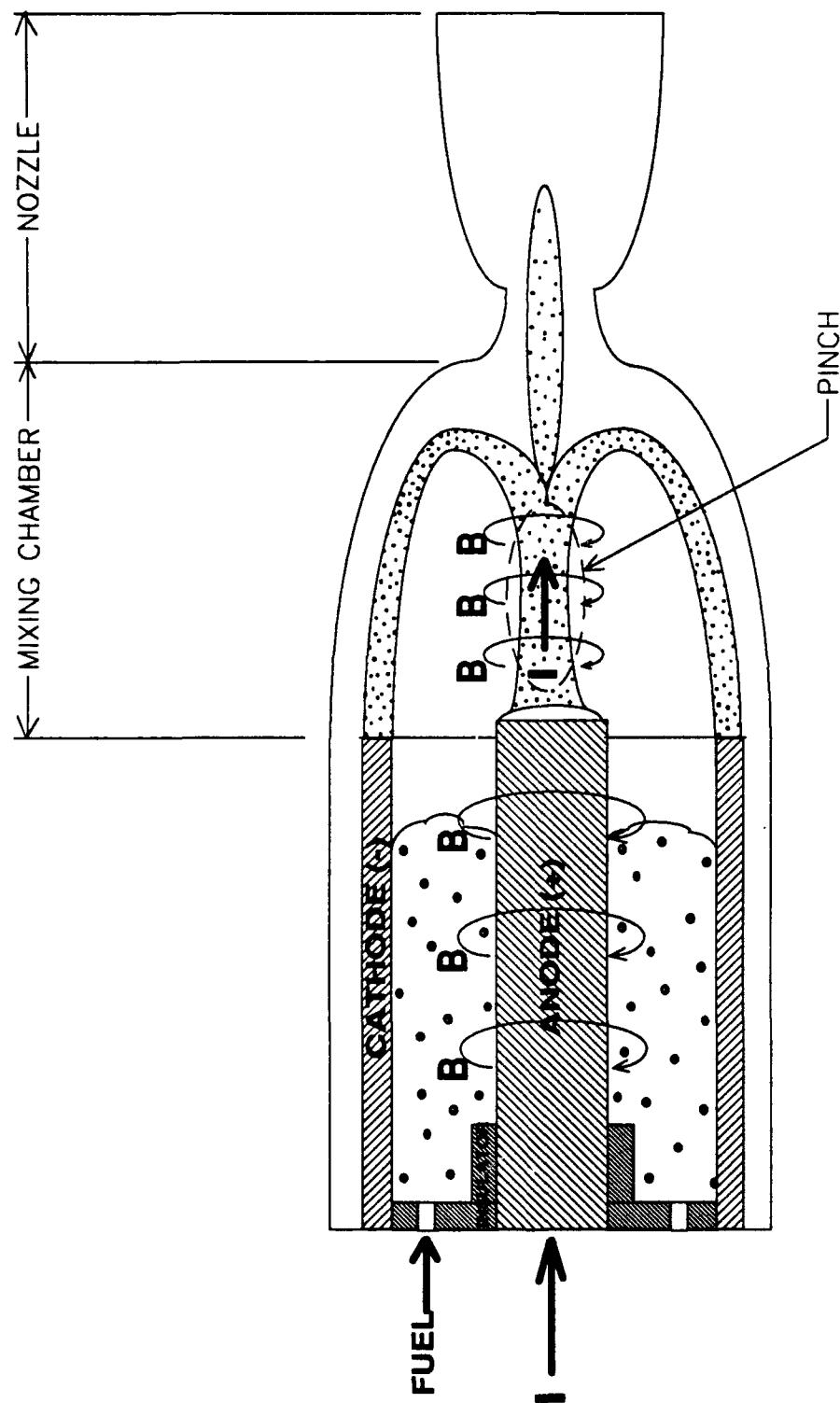


Figure 1  
Dense Plasma Focus Overview

as a "free-free" transition of an electron. In this process, a free electron is deflected by Coulomb forces due to other charged particles in its vicinity [Ref. 3]. The energy imparted to the electron during the deflection is released in the form of a photon, whose energy is governed by:

$$K_i = K_f + h\nu , \quad (1)$$

with  $K_i$  equal to the initial kinetic energy of the electron,  $K_f$  equal to its final kinetic energy, and  $h\nu$  equal to the energy released in the photon.

When the energy released in the photon approaches the kinetic energy of the emitting electron, a quantum treatment is necessary to gain a full understanding of the process [Ref. 3]. If being relativistic is considered as having a speed one-tenth that of light or greater, the energy at which electrons attain such status may be found. An electron's rest mass ( $E_{e0}$ ) of 511.0 [keV] is equal to  $m_e c^2$ . Its kinetic energy, accounting for a relativistic increase in mass, is  $(\gamma-1)m_e c^2$ . The equation for the relativistic correction factor  $\gamma$  is:

$$\gamma = \frac{1}{\sqrt{1 - (v_e^2/c^2)}} = \frac{E_{e0} + K_e}{E_{e0}} . \quad (2)$$

For  $v_e = 0.1c$ ,  $\gamma = 1.005038$ , with  $K_e = 2.574$  [keV] by Eq. (2). Because this threshold is below the temperature currently sought for operating the DPF, one must consider a relativistic treatment of the bremsstrahlung energy loss.

Further, Jackson [Ref. 4] states that a classical treatment of bremsstrahlung is only valid if:

$$\eta = (10^{-7}) \frac{Z_1 Z_2 e^2 c^2}{h\nu_1} > 1 , \quad (3)$$

where  $Z_1$  is the number of charges on the first particle (an electron, so  $Z_1 = 1$ ),  $Z_2$  is the number of charges on the second particle (a  $^3\text{He}$  ion, so  $Z_2 = 2$ ),  $e = 1.6022 \times 10^{-19}$  [C],  $h$  is Planck's constant ( $6.6262 \times 10^{-34}$  [J-s]), and  $\nu_1$  is the velocity of the first particle. For an electron of kinetic energy 2.574 [keV] ( $v_e = 0.1c$ ) as mentioned above, the corresponding speed of the electron is  $2.9979 \times 10^7$  [m/s]. Thus,  $\eta = 0.0232$ , which shows that a classical treatment is invalid, confirming the previous

relativistic assumption.

One will find, however, that a classical approach gives the correct functional dependence for most of the physical parameters [Ref. 3]. A quantum correction is often handled by a correctional coefficient known as the Gaunt factor, denoted by 'g'. This factor depends on the type of particles interacting. In a fusion plasma, the main interaction is that of electrons with the Coulomb fields of charged nuclei. For it, a value of  $g = 1.2$  is commonly used. The bremsstrahlung power density [W/m<sup>3</sup>], including Gaunt factor, is given to be [Ref. 3]:

$$P_B = 5.35 \times 10^{-37} \sqrt{K_e} n_e \sum_i n_i Z_i^2, \quad (4)$$

where  $K_e$  is the electron temperature in [keV],  $n_e$  is the number density of electrons in the plasma [m<sup>-3</sup>],  $n_i$  is the number density of ion species 'i' in the plasma [m<sup>-3</sup>], and  $Z_i$  is the atomic number of ion species 'i'.

## SYNCHROTRON RADIATION

Synchrotron radiation originates from the acceleration of charged particles as a result of the non-alignment of their velocities with their local magnetic fields. A derivation of the expression for synchrotron radiation begins with the equation for the force acting on the charge in question due to electric and magnetic fields:

$$\mathbf{F} = q(\mathbf{E} + \mathbf{v} \times \mathbf{B}). \quad (5)$$

Considering no Coulomb field  $\mathbf{E}$  to be present and the force  $\mathbf{F}$  to be given by mass times acceleration, with centripetal acceleration equal to the tangential velocity ( $\mathbf{v}_\theta$ ) squared over the radius of gyration about the  $\mathbf{B}$  field line, the resulting form of Eq. (5) is:

$$F = qv_\theta B = \frac{\gamma m v_\theta^2}{r_{gyr}}. \quad (6)$$

Vector notation has been discarded, as  $\mathbf{v}_\theta$  is orthogonal to  $\mathbf{B}$ . As stated by Tamor [Ref. 5] and as will be seen later, the synchrotron loss rate varies inversely as the cube of the particle mass. Thus, the contribution of the synchrotron radiation arising from ions may be neglected in comparison with the contribution from electrons. The frequency of emission, the cyclotron frequency, is:

$$\omega_c = \frac{v_\theta}{r_{gyr}} = \frac{qB}{\gamma m} , \quad (7)$$

and for an electron,  $q = e$ ,  $m = m_e$ , and  $\omega_c = \omega_{ce}$ .

For a charge  $q$  (here, an electron,  $q_e$ ) moving with velocity  $\mathbf{v}$  and acceleration  $\mathbf{a}$ , the total radiated power is given by:

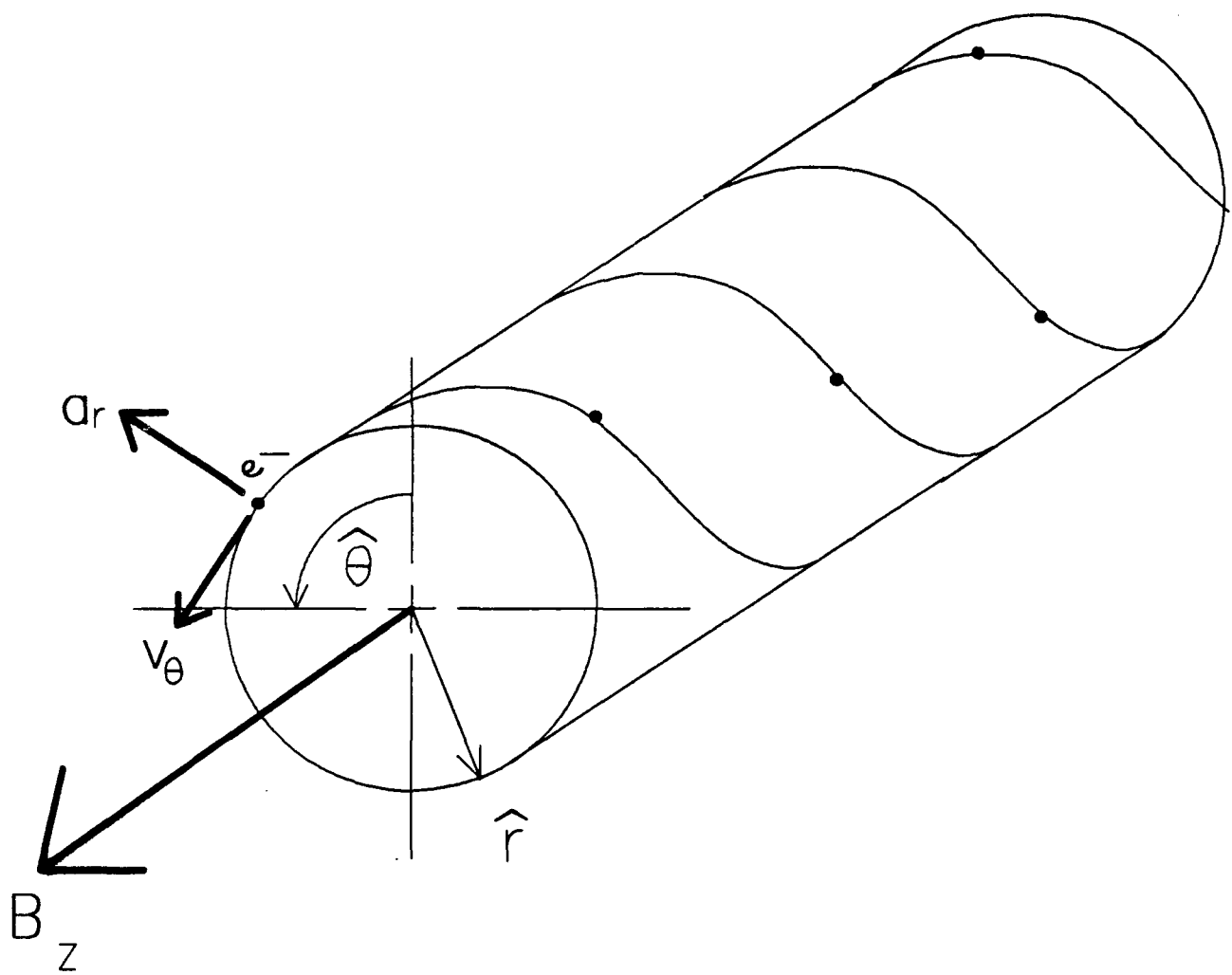
$$P_{tot} = \frac{dW}{dt} = \frac{q^2}{6\pi\epsilon_0 c} \frac{a^2 - (\mathbf{v} \times \mathbf{a}) / c^2}{(1 - \frac{v^2}{c^2})^3} , \quad (8)$$

where  $|\mathbf{a}| = |\mathbf{a}_r|$  and  $|\mathbf{a}| = |\omega_{ce} \mathbf{v}_\theta| = |(q_e B / \gamma m_e) \mathbf{v}_\theta|$  [Ref. 3]. The diagram in Figure 2 shows the locations of the vectors in the cylindrical  $(r-\theta-z)$  coordinate system being employed, assuming a nearly straight  $B$  field line. The electrons spiral about the  $B$  field lines, tracing out elliptical paths. Eq. (8) becomes:

$$P_{tot} = \frac{q_e^4 B^2 v_\theta^2}{6\pi\epsilon_0 c \gamma^2 m_e^2} . \quad (9)$$

With radial momentum  $p_r = \gamma m_e v_\theta$ , the expression for total radiation from an electron without consideration of whether it escapes follows from Eq. (9) to be:

$$P_{sync} = \frac{dW_e}{dt} = \frac{q_e^4 B^2 p_r^2}{6\pi\epsilon_0 c \gamma^4 m_e^4} . \quad (10)$$



**Figure 2**  
**Cylindrical Coordinate System**

Now,  $(p_r^2/2\omega m_e) = N_e kT_e$  and for an isotropic distribution,  $p_r^2 = (2/3)\langle p^2 \rangle$ . If  $(dW_e/dt)f(\mathbf{p})$  is now computed for Eq. (10) with  $f(\mathbf{p})$  being the **relativistic** Maxwellian distribution function, Rose and Clark [Ref. 6] have shown that:

$$P_{sync} = \frac{\mu_0 e^4 B^2}{3\pi m_e^2 c} \left( \frac{N_e kT_e}{m_e} \right) \frac{K_3(m_e c^2 / kT_e)}{K_2(m_e c^2 / kT_e)} , \quad (11)$$

where  $K_2(\dots)$  and  $K_3(\dots)$  are Modified Bessel functions of second and third order, respectively. The inverse dependency of the synchrotron radiation energy loss on the cube of the mass is now clearly evident, showing the reason for the neglect of the losses due to ion interactions.

Using the electron cyclotron radial frequency ( $\omega_{ce}$ ) defined earlier, the equation for synchrotron power emission becomes:

$$P_{sync} = \frac{\mu_0 e^2 \omega_{ce}^2}{3\pi c} \left( \frac{N_e kT_e}{m_e} \right) \left( 1 + \frac{K_e}{204.5} + \dots \right) , \quad (12)$$

of which the cyclotron radiation power ( $P_{cyc}$ ) portion comprises only the first term, that is:

$$P_{cyc} = \frac{\mu_0 e^2 \omega_{ce}^2}{3\pi c} \left( \frac{N_e kT_e}{m_e} \right) . \quad (13)$$

The units for  $K_e$  are [keV], while the units for the other terms may be found in the **GLOSSARY OF SYMBOLS**. If one assumes that relativistic effects become important when the value of  $P_{sync}$  exceeds the value of  $P_{cyc}$  by 10%, then the  $T_e$  value at which this occurs is found by:

$$P_{sync} = 1.1 P_{cyc} = P_{cyc} \left( 1 + \frac{K_e}{204.5} \right) , \quad (14)$$

which yields  $K_e \approx 20$  keV. The temperature regime being considered for the DPF model in this report is about 50 to 100 keV. One sees, then, that a relativistic treatment of this radiation type is required.

The nomenclature associated with synchrotron radiation has

become ambiguous. Technically, cyclotron and synchrotron radiation arise from the same mechanism, but their values can be quite different. Stated simply, cyclotron radiation is the non-relativistic form of the expression for synchrotron radiation emission; that is, the classical treatment. For cyclotron radiation, the frequency of emission is equal to the gyrational (cyclotron) frequency of the plasma. Synchrotron radiation, on the other hand, can be emitted at frequencies many times that of the plasma cyclotron frequency. Because the operating temperature regime of the DPF model used in this report is above the relativistic threshold, cyclotron radiation is not an appropriate name for this type of radiation. Thus, the term synchrotron radiation will be used in the remainder of this report, as it correctly applies to all energy levels of this radiation type, much as relativistic mechanics can be used to describe classical mechanical situations.

### **EFFECT OF PLASMA BETA ON SYNCHROTRON RADIATION EMISSION**

To understand the role of synchrotron radiation in modeling the DPF, the parameter  $\beta$  shall now be introduced. The  $\beta$  value of a plasma is the ratio of the particle kinetic pressure to the confining magnetic pressure at the boundary. This is given by:

$$\beta = \frac{P}{B_0^2/2\mu_0} , \quad (15)$$

where  $\mu_0$  is the permeability of free space ( $4\pi \times 10^{-7}$  [H/m]),  $P$  is the particle kinetic pressure [ $N/m^2$ ], and  $B_0$  is the magnetic induction [T] at the plasma surface. When  $\beta$  is equal to 1, a pressure balance between the expanding plasma particle pressure and the confining magnetic pressure is in existence. A value of  $\beta$  less than 1 indicates that the confining magnetic pressure must be made stronger for a given particle pressure in order to prevent pinch expansion. To re-iterate, it is assumed that a uniform constriction is present along the length (0.0254 [m]) of the pinch, giving it a cylindrical shape.

A  $\beta$  value of 1 requires the minimum magnetic induction necessary to stabilize the plasma for a given value of  $P$ .  $\beta = 1$  is a common assumption when dealing with the DPF, as modeling has shown such a  $\beta$  value to be stable, even in the presence of slight oscillations [Ref. 7]. The beneficial consequence of such a choice for  $\beta$  is the elimination of losses due to synchrotron radiation. Hulme [Ref. 8] states that for electrons having a Maxwellian distribution, the power density due to synchrotron radiation ( $P_{sync}$ ) is:



$$P_{sync} = c' \cdot B_1^2 \cdot n_e \cdot T_e , \quad (16)$$

where  $c'$  is a numerical coefficient with units  $[T^{-2} \cdot s^{-1}]$ ,  $B_1$  is the magnetic flux density within the plasma (assumed constant),  $n_e$  is the number of electrons per unit volume, and  $T_e$  is the temperature of the electrons.

Now assume that both the ions and electrons are at a temperature  $T$ . For a 50-50 D-<sup>3</sup>He mixture, the number of electrons is equal to  $1.5n_i$ , where  $n_i$  is the number of Deuterium ( $n_D$ ) and Helium-3 ( $n_{He}$ ) ions in the plasma:  $n_i = n_D + n_{He}$ . The neutral plasma pressure equation may be written as [Ref. 2]:

$$P = \frac{2}{3} \cdot (n_e + n_i) \cdot \frac{3}{2} kT = (n_e + n_i) kT , \quad (17)$$

where  $n_e$  is the electron particle density [ $m^{-3}$ ],  $n_i$  is the ion particle density [ $m^{-3}$ ], and  $(3/2)kT$  is the average charged particle three-dimensional energy [J]. If we now equate the plasma pressure to the pressure exerted on the plasma by the containing (outer) magnetic field,  $B_o$ , Eq. (17) becomes:

$$(n_e + n_i) kT = \frac{B_o^2}{2\mu_o} . \quad (18)$$

In addition a term may be added to the plasma pressure to account for any residual magnetic fields within the plasma. This term is based on the inner magnetic flux density and, when inserted, Eq. (18) becomes:

$$(n_e + n_i) kT = \frac{B_o^2 - B_1^2}{2\mu_o} . \quad (19)$$

Letting  $\beta = 1 - B_1^2/B_o^2$ , we have the expression [Ref. 8]:

$$(n_e + n_i) kT = \frac{\beta B_o^2}{2\mu_o} , \quad (20)$$

which may be compared to our original definition of  $\beta$  in Eq. (15). Eq. (18) can be obtained by letting  $\beta = 1$ , the condition in which no residual magnetic fields remain within the plasma. In terms of  $B_1$ , Eq. (20) becomes:

$$(n_e + n_i) kT = \left( \frac{\beta}{1 - \beta} \right) \frac{B_1^2}{2\mu_0} . \quad (21)$$

This expression may be solved for  $B_1^2$ , with the resulting expression substituted into Eq. (16) above to obtain:

$$P_{sync} = c' \cdot \frac{1 - \beta}{\beta} \cdot (n_e^2 + n_e n_i) T^2 . \quad (22)$$

It is now apparent what a  $\beta$  value of 1 means. The right hand side of Eq. (22) becomes 0, which means the synchrotron losses escaping the plasma fall to 0. This is an important result, for the escaping energy due to synchrotron radiation can be quite large and, thus, quite detrimental to a fusion reactor's ability to ignite and continue operating. The effect of  $\beta$  on power loss due to synchrotron radiation is further confirmed by the work of S. Tamor [Ref. 5].

Tamor presents a relation for the ratio of synchrotron power to bremsstrahlung power [Ref. 5], given by:

$$\frac{P_{sync}}{P_{brems}} = 0.3 \frac{K^{\frac{13}{4}}}{n_e^{\frac{1}{4}}} \left( \frac{1 - \beta}{\beta} \right)^{\frac{5}{4}} , \quad (23)$$

where:

$$n_e^* = 0.01 \cdot n_e \left( \frac{r_p}{1 - R} \right)^2 \frac{Z^4}{\left( 1 + \frac{\beta_i}{\beta_e} \right)^5} . \quad (24)$$

K is the temperature of the plasma [keV],  $r_p$  is the radius of the plasma [m], R is the effective reflectivity for emitted synchrotron radiation, Z is the average atomic number of the fusion fuel species, and  $\beta_i$  and  $\beta_e$  are the plasma beta values for the ions and electrons, respectively. One sees that Eq. (23) is dependent on the factor  $(1-\beta)/\beta$ . When  $\beta = 1$ ,  $P_{\text{sync}}/P_{\text{brems}} = 0$ . Thus, no synchrotron radiation is emitted for  $\beta = 1$ , agreeing with the result found in Hulme [Ref. 7].

## ABSORPTION OF SYNCHROTRON RADIATION

It is assumed in this report that the value of R is 0, which is a worst case situation. In order to see the effect of a  $\beta$  value less than 1, an arbitrary value of  $\beta = 0.9$  will be assumed. A mass density of  $2.2 \times 10^{-4}$  [kg·m<sup>-3</sup>] is used for the pinch volume of the DPF [Ref. 1]. Using Eqs. (23,24) with the parameters: R = 0, L = 1.5 [cm] (pinch radius),  $\beta = 0.9$ , Z = 1.5 (equal amounts of D and <sup>3</sup>He),  $n_D = n_{\text{He}} = 1.179 \times 10^{26}$  [m<sup>-3</sup>], and  $\beta_e = 1.5\beta_i$ ,  $P_{\text{sync}}/P_{\text{brems}}$  values for a temperature range of 10 to 100 [keV] are calculated. This is the approximate range of validity Tamor gives for Eqs. (23,24). These values are presented graphically in Figure 3. One sees that for a  $\beta$  value of 0.9, the synchrotron power escape from the pinch remains modest ( $P_{\text{sync}} < 0.1 \cdot P_{\text{brems}}$ ) until about 62 [keV]. As the temperature increases, however, reabsorption becomes increasingly important, as is evident in Figure 3. One sees that the synchrotron power is climbing significantly (as  $\text{temperature}^{13/4}$ ) as the bremsstrahlung power is increasing slowly (as  $\text{temperature}^{1/2}$ ).

In Figure 4 one sees further the effect of lowering the  $\beta$  value in the device to 0.9. Assuming the synchrotron radiation is not reabsorbed, the DPF is limited to an operating regime of approximately 34 to 117 [keV], the range in which the charged particle power exceeds the total radiative loss power. Figure 4 indicates that the optimum operating temperature would be approximately 75 [keV], at which point the charged particle power exceeds the radiative loss power by the greatest amount. It has been assumed that the equations of Tamor [Ref. 5] remain valid up to a plasma temperature of 130 [keV], in order to show the power curve intersections. Achieving a  $\beta$  value of 1 eliminates the necessity of reabsorption by preventing the escape of synchrotron radiation. It also allows one to avoid the concern for reflective walls, which is not specifically addressed in this report. In Figure 4 one notices that from about 85 keV upward, the net power gain (the difference between the  $P_{\text{brems}}$  and  $P_{\text{charged particles}}$  curves) for a  $\beta = 1$  condition remains approximately constant. A  $\beta = 1$  condition is also the most desirable approach in achieving maximum DPF performance, as is shown in the next section.

Even if the  $\beta$  value is less than 1, the plasma absorption capability is usually sufficient to achieve the 99.9% synchrotron radiation reabsorption acceptance level stated by Dawson [Ref. 9].

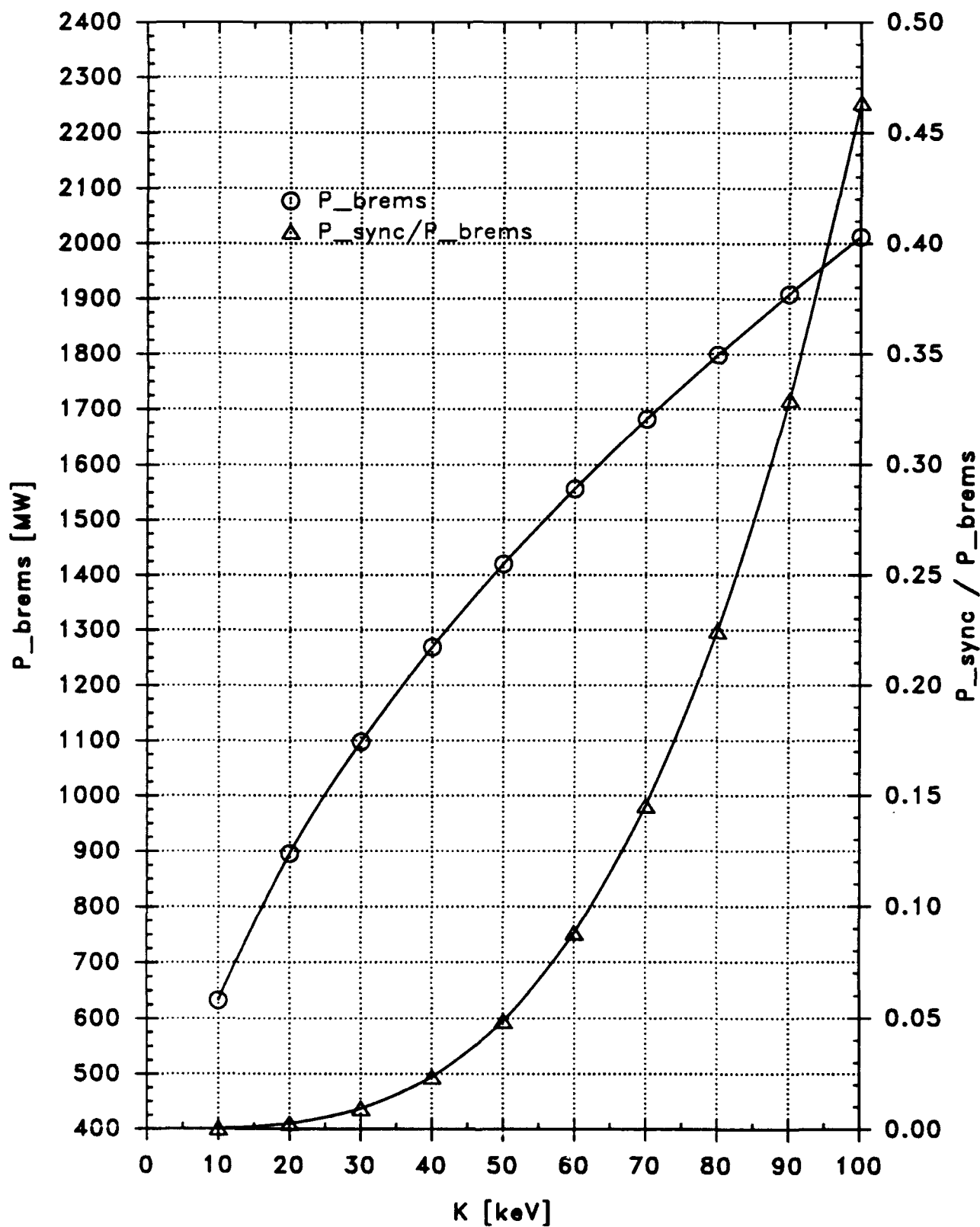


Figure 3  
 $P_{\text{brems}}$  [MW],  $P_{\text{sync}}/P_{\text{brems}}$  vs.  $K$  [keV] for  $n_e = 3.534\text{E}+26 \text{ m}^{-3}$

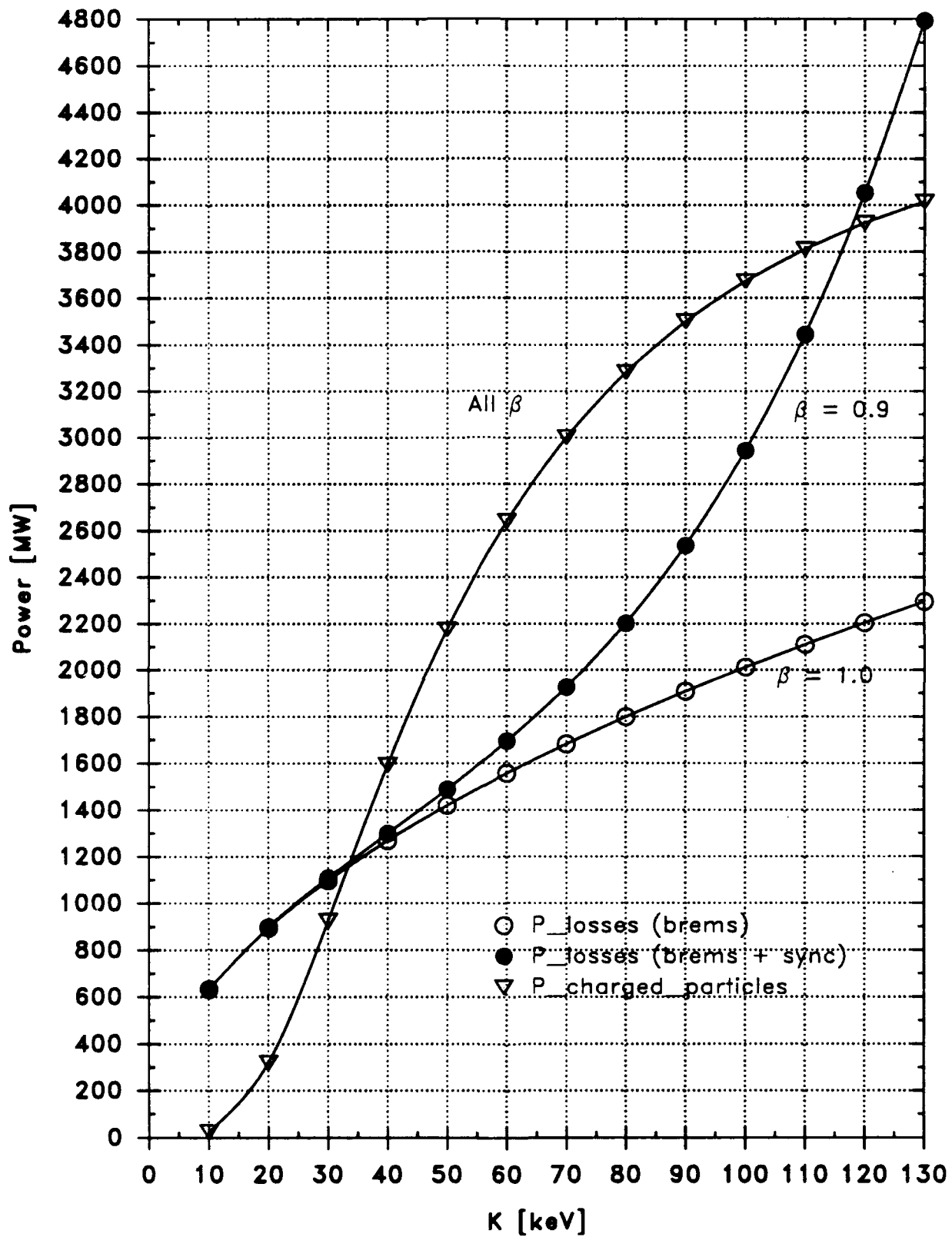


Figure 4  
Power [MW] vs. K [keV] for  $n_e = 3.534E+26 \text{ m}^{-3}$

A body that emits radiation is also an absorber of radiation. The mechanisms of emission and absorption for the transfer of radiation are associated with one another by the Kirchoff relation [Ref. 9]:

$$\frac{\text{emissivity}}{\text{absorptivity}} \propto \omega^2 T_e . \quad (25)$$

This is valid so long as the electrons in the plasma are described by a Maxwellian distribution. Here, this has been assumed to extend to the **relativistic** Maxwellian distribution assumption stated earlier. Further, the expression for the characteristic absorption length is given as [Ref. 5]:

$$\lambda_0 = \frac{c \omega_{ce}}{\omega_{pe}^2} = 1.7 \times 10^{18} \frac{B_0}{n_e} . \quad (26)$$

By substituting into Eq. (20) and considering  $n_i = (2/3)n_e$  for a 50-50 D-<sup>3</sup>He fuel mixture:

$$\beta = \frac{(1.7 \times 10^{19}) \mu_0 k T}{3 B_0 \lambda_0} , \quad (27)$$

one sees that  $\beta$  is inversely proportional to  $\lambda_0$ . Thus, a decrease in the plasma beta results in an increase in the characteristic reabsorption length.

For a typical 50-50 D-<sup>3</sup>He pinch with  $\beta = 1$ ,  $n_e = 3.534 \times 10^{26} \text{ [m}^{-3}]$ , and optimum operating temperature  $K_e = 75 \text{ [keV]}$ , a  $B_0$  value of 4217 [T] is obtained, which yields  $\lambda_0 = 2.138 \times 10^{-5} \text{ [m]}$ . The reader is reminded that a  $\beta = 1$  condition means  $B_i = 0$ , as shown earlier. A typical radius assumed for the plasma pinch is  $1.5 \times 10^{-3} \text{ [m]}$ , which is 70.2 times **greater** than  $\lambda_0$ , showing why all of the synchrotron radiation is considered to be reabsorbed within the confines of the plasma. This confirms the earlier results found using the equations in Hulme [Ref. 7] and Tamor [Ref. 5] and presents the physical basis for  $\beta = 1$  leading to  $P_{\text{sync}} = 0$ . Assuming the size of the pinch to remain relatively stable to changes in plasma temperature, magnetic induction, and particle density, the  $K_e$  value at which  $\lambda_0$  equals the plasma pinch radius may be found. This occurs when  $B/n_e = 8.824 \times 10^{-22} \text{ [T} \cdot \text{m}^3]$ . By Eq. (18), with  $n_i = (2/3)n_e$  for a 50-50 D-<sup>3</sup>He plasma, the corresponding electron temperature is based on the relation:

$$K_e = 1.315 B_o .$$

(28)

For the  $B_o$  value of 4217 [T],  $K_e = 5545$  keV, or 5.545 MeV. This value is much larger than the operational temperatures being sought after for use in the DPF [Ref. 10]. This indicates that the escape of synchrotron radiation from the dense plasma focus device operating with  $\beta = 1$  is not a major concern and is justifiably neglected.

## DENSE PLASMA FOCUS COMPUTER MODEL

### THE DENSE PLASMA FOCUS: A FUSION PROPULSION SCENARIO

The recent work of Chris Leakeas [Ref. 1] has laid the groundwork for the DPF computer model used in this study. His code was entitled, "The Dense Plasma Focus: A Fusion Propulsion Scenario." The newer model is similar in some respects to the previous model. The older code was based on a DPF system utilizing various pulsed mode operations and a D-<sup>3</sup>He fuel mixture. Currently, the new code studies the nearly steady state operation of the DPF, albeit still in a pulsed mode. As in the previous program, the mission focus was determination of the DPF's feasibility for a trip to Mars.

Leakeas' code was written with a linear program structure. A number of key assumptions about the propulsion device's operation and technological feasibility were made. These are carried over into the current model. They are:

- 1) Plasma pinch temperature scales as current squared.
- 2) Pinch dimensions are length 0.0245 [m] and radius 0.0015 [m].
- 3) Ions reach thermal equilibrium inside the pinch, allowing use of Maxwellian reaction rate parameters.
- 4) High temperature materials beyond those currently available will be developed.
- 5) Electrodes and mixing chamber (nozzle entrance area) walls can be sufficiently cooled to avoid damage.
- 6) Propellant becomes completely dissociated and ionized in the mixing chamber at 5000 K.
- 7) Capacitor technology advancement will allow factor of 10 increases in specific energy over present-day capacitors (0.2 kJ/kg) and discharging rates of 100 Hz.
- 8) Confinement times can be increased by a factor of 100 to about  $10^{-4}$  seconds.
- 9) Magnetic fields applied downstream (nozzle entrance area) from the pinch do not adversely affect the pinch formation or confinement time.

Many of the findings in Leakeas' report were quite favorable toward the DPF's use in a Mars mission. Calculations were made within the framework of the system analysis that involved the absorption and reflection of bremsstrahlung and synchrotron



radiations.  $I_{sp}$  values in the neighborhood of  $10^4$  seconds and initial thrust-to-weight ratios in the proximity of  $10^{-3}$  were calculated. The lifted payload was  $10^5$  kg.

#### **A DENSE PLASMA FOCUS MODEL (IMPROVED DPF CODE)**

The culmination of writing the new DPF code was the FORTRAN program "A Dense Plasma Focus Model." Some programming changes were made in creating the new code. For instance, many of the variable names are now mnemonic. One example is that all variables for power (energy per unit time) begin with the letter 'P' or the letters 'POW'. Also, all variables regarding time begin with 'TIM', while those for temperature begin with 'T' alone. The variable descriptions at the beginning of the code listing in Appendix A should clarify any further questions on this matter, as every variable used in the program is explained, accompanied by its units.

Program execution is governed by a set of SUBROUTINE's in the main program. One key note is that the program will stop running if, after initial calculations, it finds that the power being generated by the turbine/generator system is not sufficient to recharge the capacitor banks fully. This is a critical requirement, for the electrical current needed by the DPF for each pulse must be supplied in order for operation of the DPF to continue at the wanted keV level of the pinch.

A major concern which arose during the evolution of the current code was the fact that all cases studied showed the need for an additional power source to recharge the capacitor banks sufficiently. The reason for this finding is the result of the newer treatment given many aspects of the DPF. Some differences were minor, but others were quite detrimental to the performance of the DPF.

One area of change was the calculation of the mass of the system used to store and pump the hydrogen propellant. The original assumption was that the propellant pumping and storage system should have a mass equal to 15% of the propellant mass. This was changed to 10%, the figure used in determination of the fusion fuel pumping and storage system mass, based on the mass of the fuel (D and  $^3\text{He}$ ). This would also decrease the mass of the DPF-powered vehicle, increasing its performance.

Discussions with Dr. C. K. Choi of Purdue [Ref. 11] led to including the D-D side reactions in the power density and electromagnetic radiation loss calculations. He felt that for the short duration ( $10^{-4}$  seconds) of the pinch, the D-D reactions would be significant, but secondary reactions such as D-T would not be. Higher power densities can be achieved with increased amounts of  $^3\text{He}$  in the fuel mix, but at a cost of increased fuel mass and

increased bremsstrahlung radiation loss due to its higher atomic number compared with D. Before many of the major changes to the code, the optimum fuel mix based on a maximum  $I_{sp}$  value was found to be approximately 60% D and 40%  $^3\text{He}$ .

The first major alteration to the original DPF code dealt with the usage of the total efficiency (EFFTOT) of the pinch device. The input file (see Appendix B) for the program contains two efficiency terms, each corresponding to a fraction of particles which are trapped. The efficiency of the current sheath, the snowplow, (EFFPLW) is the fraction of particles which are swept up in the current sheath discharge as it runs the length of the anode as seen in Figure 5. The pinch efficiency (EFFPIN) is the fraction of particles that are caught in the snowplow effect and then trapped in the plasma pinch at the tip of the anode. Thus,

$$\text{EFFTOT} = \text{EFFPLW} \times \text{EFFPIN} . \quad (29)$$

EFFTOT is used in the calculation of the particle number density in the pinch. In the original program, it was further included in the calculations for the magnetic induction at the pinch surface (BPINCH), the bremsstrahlung power loss, and the cyclotron power loss. In the new code, the EFFTOT term is only used in the calculation of the number of particles in the pinch.

The reason for including EFFTOT only once in the calculations is that its value propagates through the program. The number density in the pinch is used to calculate BSURF (called BPINCH in Leakeas' program), which in turn is used to calculate the needed current (IMAX) in the pinch. EFFTOT is also propagated through the calculations of the bremsstrahlung and synchrotron radiation power losses. In the original code the total efficiency was included in these same expressions that already included either BPINCH or NPNCH. Since EFFTOT is 0.175 using EFFPLW = 0.70 and EFFPIN = 0.25, the additional inclusion of EFFTOT into an expression would decrease the actual value of a particular term by a factor of 5.71. This was found to affect the performance of the DPF considerably.

The condition  $\beta = 1$ , as mentioned earlier in the previous section, formed the basis for the next major change in the code. Study of the original program following changes due to EFFTOT's essentially multiple inclusion in some equations showed that the calculated BSURF and number densities resulted in  $\beta \approx 2.46$ . This would indicate that the plasma pressure on the inside of pinch is about two and a half times greater than that of the magnetic pressure on the outside. Such a plasma is unstable, as it is not confined, but is expanding outward. Thus, the needed number densities in the pinch cannot be obtained.

Therefore, the new code is arranged so  $\beta = 1$  is a condition

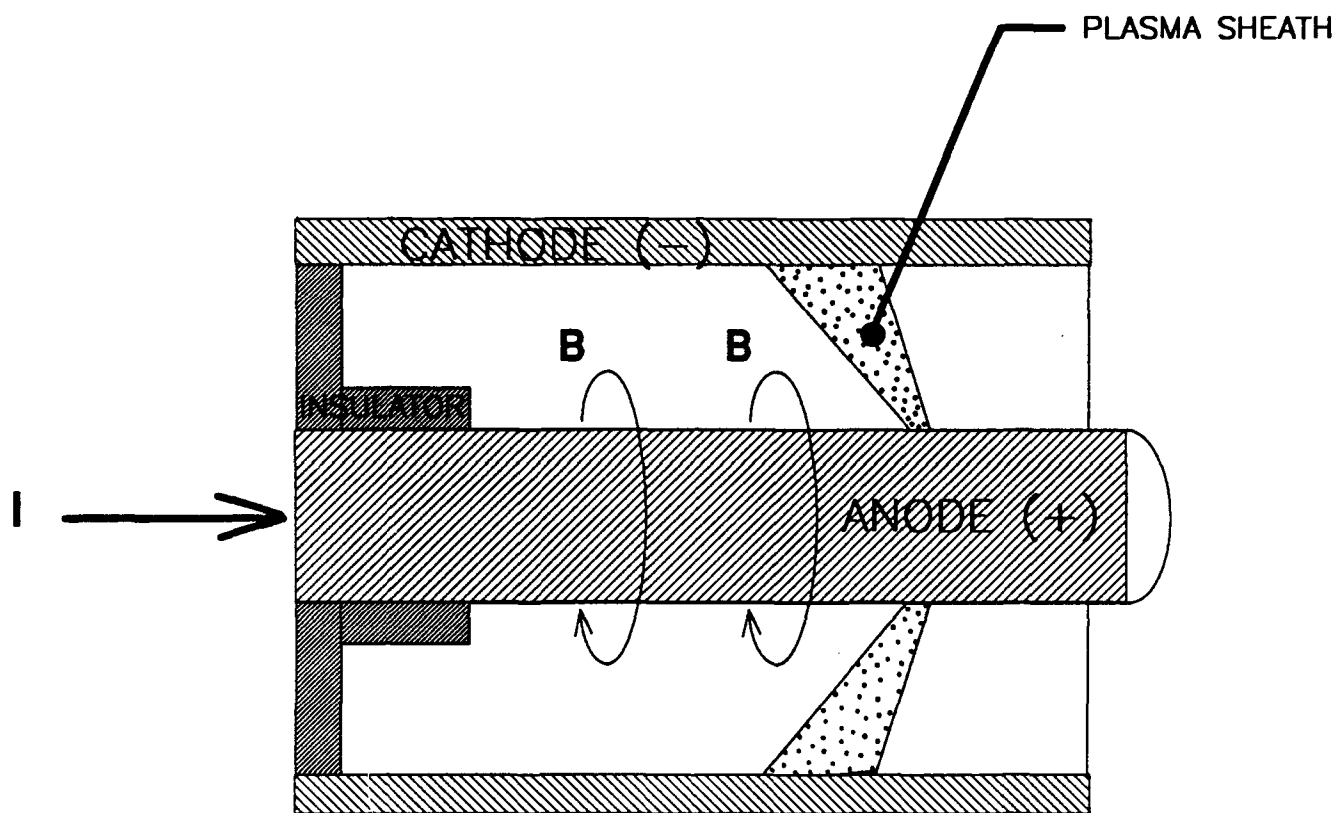


Figure 5  
Plasma Sheath ("Snowplow") Rundown

inherent to the calculations in the program. This is done in a straightforward manner. The initial ion density in the pinch is given by:

$$n_{beg} = \frac{\eta_{tot} \rho_{init} V_{init}}{m_{av} V_{pin}} , \quad (30)$$

where  $n_{beg}$  is the number of ions in the pinch in the beginning,  $\rho_{init}$  is the initial mass density of the fuel particles in the entrance of the DPF,  $V_{init}$  is the initial volume of the focus chamber,  $m_{av}$  is the average mass of each particle within the pinch, and  $V_{pin}$  is the volume of the pinch itself. From  $n_{beg}$  the number densities of the D and  $^3\text{He}$  particles and the electrons within the pinch are determined. Then, using Eq. (20) with  $\beta = 1$ , the magnetic induction at the surface,  $B_0$ , is found.  $B_0$  is used to obtain the value for IMAX ( $I_{max}$ ) needed in order to create a magnetic field strong enough to contain the calculated number density of the pinch particles. The result is that a much higher current than previously thought necessary is required. Therefore, a larger amount of power is required of the capacitor bank in order to supply the needed current. One finds that the turbine/generator system used, with a turbine efficiency of 100% and a generator efficiency of 30%, is not able to meet this requirement.

Some additional concerns were also addressed in writing the new code. The central focus of the remaining changes was the treatment of synchrotron radiation losses. The original program used a variable CYCREFL for the fraction of emitted synchrotron energy reflected into the plasma, assuming a value of ranging from 0.6 to 0.8. As seen earlier in this report in **EFFECT OF BETA ON SYNCHROTRON RADIATION**, such a value is too costly for operation of the DPF. It was shown earlier how much just letting  $\beta = 0.9$  can affect the losses due to synchrotron radiation emissions. The earlier code, with its  $\beta$  value of  $\sim 2.46$ , means that no synchrotron radiation could have been produced. As seen earlier, no synchrotron radiation emission occurs for a  $\beta$  value  $\geq 1$ . This indicates a CYCREFL value of 1.00. This variable is renamed REFSYN in the new program. It is retained in the program because in the future, modeling the DPF with a  $\beta$  value less than 1.00 may be desirable. In such a case, synchrotron radiation would possibly escape the plasma, causing one to be concerned about how much of it is reflected into the plasma by the chamber walls and nozzle entrance. Until such a time, though, the value of REFSYN has been assigned a value of 1.00.

When research began on this project, it was thought that the amount of synchrotron radiation reflected into the plasma could be calculated using the plasma depth  $D_p$  [Ref. 2]:

$$D_p = \frac{2n_e r_p}{B^{\frac{3}{2}} \eta^{\frac{1}{2}} (1 - \beta)^{\frac{3}{4}}} , \quad (31)$$

where  $r_p$  is the plasma radius [m],  $\eta$  is the electrical resistivity [ $\Omega$ -m] of the reflecting material at a specified temperature, and other variables are as noted earlier. The reflecting materials were the electrodes, assumed to be made of copper. After discovering that the device had a  $\beta$  value  $\geq 1$ , it was decided to forego such calculations, as  $\beta = 1$  represents the ideal condition for operating this fusion device. Further, it would require much more time and detail to determine how much of the synchrotron radiation emissions (which only occurs for  $\beta < 1$ ) are reflected. Since the pinch is fully outside the DPF device, it is unlikely any appreciable emissions would be reflected to the pinch. The mixing chamber (nozzle entrance vessel) walls may be able to reflect some of the emissions, but this would only seem to be appreciable if the chamber size is close to that of the focus device. One would need to study how reflection is affected by distance from the pinch and how synchrotron radiation is affected by transmission through the propellant flow leaving the chamber and entering the nozzle.

One final area of concern was the cooling of the electrodes, which are heated ohmically during the focus process. It was decided that deuterium gas could be pumped through a hollow anode and possibly the annulus containing the snowplow between pulses. The mass of deuterium necessary to absorb all of the Ohmic power being produced in a single pulse is given by:

$$\dot{m}_D = \frac{P_{ohm} \Delta t}{c_{p,D} \Delta T} \text{ [kg/s]} , \quad (32)$$

where  $P_{ohm}$  is the Ohmic heating power [W],  $\Delta t$  is the length of time in one pulse [s],  $c_{p,D}$  is the specific heat of deuterium, and  $\Delta T$  is the change in temperature [ $^{\circ}$ K] of the electrodes during one pulse. Since the repetition rate used in the DPF model is 100 times per second, this mass of deuterium is multiplied by a factor of 100 to determine mass flow per second.

## CONCLUSIONS AND RECOMMENDATIONS

This report has brought forward the benefit of having a dense plasma focus with  $\beta = 1$ : no escape of emitted synchrotron radiation from the plasma. In fact, an appreciable electron temperature must be attained before the characteristic synchrotron radiation reabsorption length exceeds the radius of the plasma pinch. It would be of possible value in the future to determine if the exact shape of the plasma pinch would have any effect on the characteristic reabsorption length,  $\lambda_0$ . Further work in this area would validate whether the value of  $\lambda_0$  in relation to the pinch radius is a sufficient indicator of whether synchrotron radiation escapes the plasma. For the time being, however, neglecting the energy losses due to the escape of synchrotron radiation is appropriate for a  $\beta = 1$  device.

The synchrotron radiation relation to the plasma beta is incorporated into the dense plasma focus code listed in Appendix A. The new DPF code is modularized, so as new facets are added to it, their incorporation is made simple through the insertion of new subroutines. Slight corrections have been made to the original code, as well as updating it with new information. Some test input files have been run, with the highest resulting  $I_{sp}$  values in the range 900-1000 seconds. The maximum Thrust-to-Weight ratios have been on the order of  $10^{-4}$ . These values are not as favorable as those listed in the earlier report by Leakeas [Ref. 1]. The current code is based on the code listing found at the back of Leakeas' report. The findings in the earlier report may have been based on a DPF device working in a pulsed manner, rather than the near steady-state conditions on which the code herein is based. Recently, it has been found that a pulsed mode offers higher performance [Ref. 12]. In the future, it would be advisable to update the code to working in a pulsed manner and compare the results to the current, near steady-state device.

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## APPENDIX A

```

C #####
C ##### A DENSE PLASMA FOCUS MODEL #####
C #####

```

```

C PROGRAMMER:  Larry T. Cox, Jr.
C DATE:        4 JUN 1991
C LATEST REVISION:  12 AUG 1991

```

```

C AFFILIATIONS:  School of Nuclear Engineering
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C
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```

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C Based on the FORTRAN code "THE DENSE PLASMA FOCUS:  A FUSION PROPULSION
C SCENARIO" by Christopher L. Leakeas

```

### VARIABLE DESCRIPTIONS

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C IMPLICIT REAL*8 (Static)

```

Variable	Description
=====	=====
BOLTZ	Boltzmann constant ( $1.381 \times 10^{-23}$ J/K)
CPE	Specific energy of an electron
CPH2	" " of diatomic hydrogen
CPHION	" " of a hydrogen atom
FSYNAB	Fraction of synchrotron radiation absorbed in the walls and electrodes
GRAV	Acceleration due to gravity ( $9.80 \text{ m/s}^2$ )
IMAGNT	Current flowing in the magnet at the nozzle entrance
MH2	Atomic mass of diatomic hydrogen
MUNAUT	Permittivity of space ( $4 \times \pi \times 10^{-7} \text{ H/m}$ )
PI	Pi (3.1415)
RHOCU	Mass density of copper
TDISON	Time of dissociation and ionization of diatomic hydrogen
TIMDIS	Time of dissociation for diatomic hydrogen
WDDN	Energy (work) released in one D(d,n)3He reaction
WDDP	" " " in one D(d,p)T reaction
WDH3	" " " in one 3He(d,p)4He reaction
DDN1-DDN4	Coefficients for calculation of DDn rx rate parameter
DDP1-DDP4	" " " of DDp rx rate parameter
DH31-DH34	" " " of D-3He rx rate parameter

C IMPLICIT REAL\*8 (Dynamic)

Variable	Description
=====	=====
BETA	Plasma pressure balance coefficient (= 1.00)
BSURF	Magnetic induction at surface of plasma pinch (Tesla)
COEFF	Coefficient
CPCAP	Specific energy of the capacitors
DAYS	Number of days the mission is to last
ECAP	Stored energy in capacitors required to supply needed current IMAX
EFFTOT	Total efficiency of focus device (fraction of injected particles which end up in the pinch)
EFFPLW	Efficiency of the snowplow (fraction of injected particles caught in the snowplow)
EFFPIN	Efficiency of the pinch (fraction of snowplow particles trapped in the pinch)
REFSYN	Reflected fraction of synchrotron radiation back into the plasma pinch due to chamber wall reflection
REFBRE	Reflected fraction of Bremsstrahlung radiation back into the plasma pinch by means of seeding the propellant and/or fuel
FLOW	Mass flow of hydrogen into the turbine (kg/s)
FLOWAD	Mass flow of hydrogen in excess of that needed to generate the required power to charge the capacitor bank (kg/s)
FLOWD	Mass flow of deuterium coolant through the electrodes (kg/s)
FLOMOR	Mass flow of hydrogen added to FLOW (kg/s)
FARADS	Capacitance of the capacitor bank (Farads)
FFBURN	Fraction of original D and 3He number densities consumed during the burn
FRACD	Fraction of deuterium in the fuel mixture initially
FRACH3	Fraction of Helium-3 in the fuel mixture initially
IDOT	Time rate of change of the capacitor bank current (Amps/second)
IMAX	Current required for operation of the pinch device (Amps)
IMAXSQ	IMAX**2
INDIN	Initial inductance of the capacitor bank (Henries)
ISP	Specific impulse of the DPF system (seconds)
KTPIN	Kinetic temperature in the pinch (keV)
KTLOG	Log base 10 of KTPIN used in rx rate parameter calculations
LANODE	Length of the anode (m)
LDOT	Time rate of change of the capacitor bank inductance (Henries/s)
LPINCH	Length of the plasma pinch (m)
MASSAV	Average mass of each atom and/or ion in the pinch (
MCAP	Mass of the capacitor bank (kg)
MFUEL	Mass of the D and 3He fuel carried (kg)
MFUSYS	Mass of the pumping and storage system for the fuel (kg)
MMAGNT	Mass of the magnet used in the magnetic nozzle (kg)
MPAYLD	Mass of the payload (kg)
MPROP	Mass of the hydrogen propellant carried (kg)
MPROST	Mass of the propellant pumping and storage system (kg)

C	MSHELD	Mass of the LiH neutron shield (kg)
C	MTOTAL	Total mass of all DPF system components (kg)
C	NBEG	Number density of all atoms and/or ions in the pinch at the beginning ( $\#/m^{**3}$ )
C	NDBEG	Number density of deuterium atoms/ions in the pinch at the beginning ( $\#/m^{**3}$ )
C	NH3BEG	Number density of Helium-3 atoms/ions in the pinch at the beginning ( $\#/m^{**3}$ )
C	NPBEG	Number density of hydrogen/protons in the pinch at the beginning ( $\#/m^{**3}$ )
C	NTBEG	Number density of tritium/tritons in the pinch at the beginning ( $\#/m^{**3}$ )
C	NH4BEG	Number density of Helium-4/alphas in the pinch at the beginning ( $\#/m^{**3}$ )
C	PABSW	Waste power absorbed in the walls (MW)
C	PBREM	Bremsstrahlung power produced (MW)
C	PCAPAD	Additional power needed to charge capacitor bank (MW)
C	PDELTA	Change in power equal to fusion power produced minus the electromagnetic radiation power losses in one time interval (MW)
C	PDELTO	Overall change in power over one pulse (MW)
C	PFDDN	Fusion power produced from DDn in one time interval (MW)
C	PFDDP	Fusion power produced from DDp in one time interval (MW)
C	PFDH3	Fusion power produced from D-3He in one time interval (MW)
C	PFT	Total fusion power produced in one time interval (MW)
C	PFTDDN	Total fusion power produced by DDn in one pulse (MW)
C	PFTDDP	Total fusion power produced by DDp in one pulse (MW)
C	PFTDH3	Total fusion power produced by D-3He in one pulse (MW)
C	PFTOT	Total fusion power produced in one pulse (MW)
C	PLEFT	Power remaining after propellant is fully ionized to heat the ionized propellant (MW)
C	PLOSS	Power lost to electromagnetic radiation during each time interval (MW)
C	PLOSTO	Overall power lost to electromagnetic radiation during the length of one pulse (MW)
C	PMAGNT	Power emitted by the magnet at the nozzle entrance (MW)
C	POHMIC	Power absorbed in the copper electrodes as a result of ohmic heating (MW)
C	POWELC	Electrical power generated by the heated hydrogen gas to charge the capacitor banks (MW)
C	POWIN	Input power needed by the focus to fire (MW)
C	PSYNC	Synchrotron power emitted during the current time interval (MW)
C	PTBREM	Total bremsstrahlung power loss during one pulse (MW)
C	PTDIS	Total power used to heat and dissociate the hydrogen propellant (MW)
C	PTSYNC	Total synchrotron power loss during one pulse (MW)
C	PWASTE	Total waste power derived from absorbed bremsstrahlung, absorbed synchrotron radiation, and magnet power used to heat the hydrogen propellant before entering the turbine (MW)
C	RADPIN	Radius of the plasma pinch (m)

```

C      RANODE      Outer radius of the focus anode (m)
C      RATREP      Rate of repetition of the focus device (pulses/s)
C      RCATH       Inner radius of the focus cathode (m)
C      RHOIN       Initial mass density of the fuel at the entrance to the
C                  focus device (kg/m**3)
C      RRDDN       Current reaction rate for D(d,n)3He (reactions/m**3-s)
C      RRDDP       Current reaction rate for D(d,p)T (reactions/m**3-s)
C      RRDH3       Current reaction rate for 3He(d,p)4He (reactions/m**3-s)
C      RRDDN1      Initial reaction rate for D(d,n)3He (reactions/m**3-s)
C      RRDDP1      Initial reaction rate for D(d,p)T (reactions/m**3-s)
C      RRDH31      Initial reaction rate for 3He(d,p)4He (reactions/m**3-s)
C      SVDDN       Reaction rate parameter for DDn (m**3/s)
C      SVDDP       Reaction rate parameter for DDp (m**3/s)
C      SVDH3       Reaction rate parameter for D-3He (m**3/s)
C      TAV         Average temperature of propellant in the chamber (K)
C      THICK       Thickness of the LiH neutron shield (m)
C      THRE        Thrust due to electrons (N)
C      THRION      Thrust due to ions (N)
C      THRNP       Thrust due to non-pinch gases (N)
C      THRPRO      Thrust due to propellant (N)
C      THRTOT      Total thrust (N)
C      TIMPIN      Duration of a pinch (s)
C      TSTAG       Stagnation temperature of the propellant (K)
C      TTHROT      Temperature of the throat (K)
C      VEXE        Exhaust velocity of the electrons (m/s)
C      VEXI        Exhaust velocity of the ions (m/s)
C      VOLTS       Voltage of the capacitor bank (V)
C      VTHROE      Throat velocity of the electrons (m/s)
C      VTHROI      Throat velocity of the ions (m/s)
C      VOLINI      Initial volume of the fuel in annular region between the
C                  anode and the cathode (m**3)
C      VOLPIN      Volume of the pinch (m**3)
C      XSAREA      Cross-sectional area of the annular region between the
C                  anode and the cathode (m**2)
C      ZEFF        (Effective atomic number) x (number of particles in the
C                  pinch)**2
C
C23456

```

```
PROGRAM DPF
```

```
IMPLICIT REAL*8 (A - Z)
```

```
INTEGER I, J, K, STEPS, RUNTYP
```

```
INCLUDE 'BLOCK.DPF'
```

```

DATA NDBEG, NH3BEG, NH4BEG, NPBEG, NTBEG /0.0,0.0,0.0,0.0,0.0/
DATA NPIND, NPINH3, NPINH4, NPINP, NPINT /0.0,0.0,0.0,0.0,0.0/
DATA NPINCH, NPINEL, PDELTA /0.0,0.0,0.0/
DATA PFDH3, PFDDN, PFDDP, PFT, PFTOT /0.0,0.0,0.0,0.0,0.0/
DATA PFTDDN, PFTDDP, PFTDH3 /0.0,0.0,0.0/

```

```

DATA PTBREM,PTSYNC,PTLOSS /0.0,0.0,0.0/
DATA PDELTO,PLOSTO /0.0,0.0/
DATA PBREM,PLOSS,PLOSS /0.0,0.0,0.0/
DATA RRDDN,RRDDP,RRDH3 /0.0,0.0,0.0/
DATA ZEFF /0.0/

```

```

CALL INPUTDAT(STEPS,RUNTYP)
CALL FOCUSPAR
CALL REACRATE
CALL PULSE(STEPS)
CALL POWER(STEPS)

```

```

IF ((POWIN+PMAGNT+ECAP).GT.POWELC) THEN
  PRINT *, 'Not enough energy derived from waste heat to'
  PRINT *, 'charge the capacitor bank!!!'
ELSE
  CALL MARSSYS(STEPS)
  CALL RESULTS(STEPS)
ENDIF

```

```

STOP
END

```

---

# BLOCK DATA ASSIGNMENT

---

## BLOCK DATA BLKDAT

```

REAL*8 BOLTZ,CPE,CPH2,CPHION,FSYNAB,GRAV,IMAGNT,MH2,MUNAUT,PI
REAL*8 RHOCU,TDISON,TIMDIS,WDDN,WDDP,WDH3
REAL*8 DDN1,DDN2,DDN3,DDN4,DDP1,DDP2,DDP3,DDP4,DH31,DH32,DH33,DH34

```

```

COMMON /CONST1/ BOLTZ,CPE,CPH2,CPHION,FSYNAB,GRAV,IMAGNT,MH2,
& MUNAUT,PI,RHOCU,TDISON,TIMDIS,
& WDDN,WDDP,WDH3
COMMON /CONST2/ DDN1,DDN2,DDN3,DDN4,DDP1,DDP2,DDP3,DDP4,
& DH31,DH32,DH33,DH34

```

```

DATA BOLTZ,CPE,CPH2,CPHION /1.38E-23,1.517E07,4157.0,8267.0/
DATA FSYNAB,GRAV,IMAGNT,MH2 /0.2,9.807,3.18E05,3.34E-27/
DATA MUNAUT,PI,RHOCU /1.257E-06,3.1415,1.673E-08/
DATA TDISON,TIMDIS /5000.0,1.0E-07/
DATA WDDN,WDDP,WDH3 /5.24E-13,6.46E-13,2.93E-12/
DATA DDN1,DDN2,DDN3,DDN4 /0.29811,-2.08296,5.70135,-22.08780/
DATA DDP1,DDP2,DDP3,DDP4 /0.30795,-2.12009,5.68718,-22.03746/
DATA DH31,DH32,DH33,DH34 /0.35715,-3.32451,10.11363,-25.66533/
END

```

# INPUT DATA

---

```
SUBROUTINE INPUTDAT(STEPS,RUNTYP)

IMPLICIT REAL*8 (A - Z)

INTEGER STEPS,RUNTYP

CHARACTER*12 INFILE

INCLUDE 'BLOCK.DPF'

OPEN (UNIT=1,FILE='MARS.DAT',STATUS='OLD',FORM='FORMATTED',
&      ACCESS='SEQUENTIAL')

READ (1,*)
READ (1,*) REFSYN
READ (1,*)
READ (1,*) RANODE
READ (1,*)
READ (1,*) RCATH
READ (1,*)
READ (1,*) LANODE
READ (1,*)
READ (1,*) RHOIN
READ (1,*)
READ (1,*) REFBRE
READ (1,*)
READ (1,*) INDIN
READ (1,*)
READ (1,*) FARADS
READ (1,*)
READ (1,*) CPCAP
READ (1,*)
READ (1,*) EFFPLW
READ (1,*)
READ (1,*) EFFPIN
READ (1,*)
READ (1,*) LPINCH
READ (1,*)
READ (1,*) RADPIN
READ (1,*)
READ (1,*) FRACD
READ (1,*)
READ (1,*) FRACH3
READ (1,*)
READ (1,*) KTPIN
READ (1,*)
READ (1,*) RATREP
```

```

READ (1,*)
READ (1,*) TIMPIN
READ (1,*)
READ (1,*) STEPS
READ (1,*)
READ (1,*) DAYS

```

```

CLOSE (UNIT=1)

```

```

RETURN
END

```

---

### FOCUS PARAMETER CALCULATIONS

---

```

SUBROUTINE FOCUSPAR

```

```

IMPLICIT REAL*8 (A - Z)

```

```

INCLUDE 'BLOCK.DPF'

```

```

XSAREA = PI * (RCATH**2 - RANODE**2)  ![m**2]
VOLINI = XSAREA * LANODE  ![m**3]
VOLPIN = PI * RADPIN**2 * LPINCH  ![m**3]
EFFTOT = EFFPLW * EFFPIN
MASSAV = FRACH3*5.00E-27 + FRACD*3.34E-27  ![kg]

```

Calculate Initial Pinch Number Density

```

NBEG = EFFTOT * RHOIN * VOLINI / (MASSAV*VOLPIN)  ![#/m**3]
NDBEG = FRACD * NBEG  ![#/m**3]
NH3BEG = FRACH3 * NBEG  ![#/m**3]

```

Use beta value of 1.0 so that magnetic and particle pressures balance at the plasma's surface and then calculate the magnetic induction B at the surface

```

BSURF = SQRT( (2.0*NDBEG+3.0*NH3BEG)*1.6E-16*KTPIN*2.0*MUNAUT )  ![T]

```

Needed Current is obtained using Ampere's Law

```

IMAX = BSURF*2.0*PI*RADPIN / MUNAUT
IMAXSQ = IMAX**2

```

```

NTBEG = 0.0
NPBEG = 0.0
NH4BEG = 0.0

```

```

RETURN
END

```

=====

REACTION RATE PARAMETER CALCULATIONS

=====

SUBROUTINE REACRATE

IMPLICIT REAL\*8 (A - Z)

INCLUDE 'BLOCK.DPF'

KTLOG = LOG10(KTPIN)  
 SVDH3 = 1.0E-06 \* 10\*\*(DH31\*(KTLOG\*\*3) + DH32\*(KTLOG\*\*2) +  
 & DH33\*KTLOG + DH34) ![m\*\*3/s]  
 SVDDN = 1.0E-06 \* 10\*\*(DDN1\*(KTLOG\*\*3) + DDN2\*(KTLOG\*\*2) +  
 & DDN3\*KTLOG + DDN4) ![m\*\*3/s]  
 SVDDP = 1.0E-06 \* 10\*\*(DDP1\*(KTLOG\*\*3) + DDP2\*(KTLOG\*\*2) +  
 & DDP3\*KTLOG + DDP4) ![m\*\*3/s]  
 RETURN  
 END

=====

PULSE CALCULATIONS

=====

SUBROUTINE PULSE(STEPS)

IMPLICIT REAL\*8 (A - Z)

INTEGER I,J,K,STEPS

INCLUDE 'BLOCK.DPF'

NPIND = NDBEG  
 NPINH3 = NH3BEG  
 NPINP = NPBEG  
 NPINH4 = NH4BEG

J = 0  
 PRINT \*

DO 235 I = 1,STEPS-1  
 J = J + 1  
 IF (J.EQ.500) THEN  
 220 PRINT 220, 'STEP #',I  
 FORMAT ('+',A,I6)  
 J = 0  
 ENDIF



```

RRDH3 = NPIND * (NPINH3*SVDH3)  ![m**-3*s**-1]
RRDDN = 0.25 * NPIND * (NPIND*SVDDN)  ![m**-3*s**-1]
RRDDP = 0.25 * NPIND * (NPIND*SVDDP)  ![m**-3*s**-1]

```

C  
C Charged particle fusion power from pinch  
C

```

& PFDH3 = RRDH3*WDH3*VOLPIN*RATREP*TIMPIN*
      (1.0/STEPS)*1.0E-06  ![MW]
& PFDDN = 0.25*RRDDN*WDDN*VOLPIN*RATREP*TIMPIN*
      (1.0/STEPS)*1.0E-06  ![MW]
& PFDDP = RRDDP*WDDP*VOLPIN*RATREP*TIMPIN*
      (1.0/STEPS)*1.0E-06  ![MW]
& PFTOT = PFDH3 + PFDDN + PFDDP  ![MW]

NPINEL = NPIND+2.0*NPINH3+NPINT+2.0*NPINH4+NPINP
ZEFF = (NPIND+4.0*NPINH3+NPINT+4.0*NPINH4+NPINP)

PSYNC = 6.21E-17 * VOLPIN * BSURF**2 * NPINEL *
& KTPIN * (1.0+KTPIN/204.5) * RATREP * TIMPIN *
& (1.0/STEPS) * 1.0E-06  ![MW]
PBREM = 5.35E-37 * NPINEL *  ![* EFFTOT**2]
& SQRT(KTPIN) * VOLPIN * ZEFF *
& RATREP * TIMPIN * (1.0/STEPS) * 1.0E-06  ![MW]
PLOSS = (1.0-REFBRE)*PBREM + (1.0-REFSYN)*PSYNC  ![MW]
PDELTA = PFTOT - PLOSS  ![MW]

```

C  
C Add the terms of the steps to the power values  
C

```

PFT = PFT + PFTOT  ![MW]
PLOSTO = PLOSTO + PLOSS  ![MW]
PDELTO = PDELTO + PDELTA  ![MW]
PFTDH3 = PFTDH3 + PFDH3  ![MW]
PFTDDN = PFTDDN + PFDDN  ![MW]
PFTDDP = PFTDDP + PFDDP  ![MW]
PTBREM = PTBREM + PBREM  ![MW]
PTSINC = PTSINC + PSYNC  ![MW]

```

```

IF (I.EQ.1) THEN
  RRDH31 = RRDH3
  RRDDN1 = RRDDN
  RRDDP1 = RRDDP
ENDIF

```

```

& NPIND = NPIND - (RRDH3 + 2.0*RRDDN+
      2.0*RRDDP) * TIMPIN * (1.0/STEPS)  ![#/m**3]
& NPINH3 = NPINH3 - (RRDH3 - RRDDN) * TIMPIN *
      (1.0/STEPS)  ![#/m**3]
& NPINT = NPINT + RRDDP * TIMPIN * (1.0/STEPS)  ![#/m**3]
NPINH4 = NPINH4 + RRDH3 * TIMPIN * (1.0/STEPS)  ![#/m**3]
NPINP = NPINP + (RRDH3 + RRDDP) * TIMPIN * (1.0/STEPS)  ![#/m**3]
NPINCH = NPIND + NPINH3 + NPINT + NPINH4 + NPINP  ![#/m**3]

```

235 CONTINUE

RETURN  
END

```
C=====
C
C                               POWER CALCULATIONS
C=====
```

SUBROUTINE POWER(STEPS)

IMPLICIT REAL\*8 (A - Z)

INTEGER I,J,K,STEPS

INCLUDE 'BLOCK.DPF'

C  
C Stored electrical energy needed in capacitor = ECAP

ECAP = 0.5 \* INDIN \* (IMAX+IMAGNT)\*\*2 \* 1.0E-06 ![MJ]

C  
C Determine voltage of capacitor bank from ECAP value and FARADS value

VOLTS = SQRT(2.0\*ECAP\*1.0E+06/FARADS)

POWIN = VOLTS\*(IMAGNT+IMAX)\*RATREP\*TIMDIS\*1.0E-06 ![MW]

C  
C Synchrotron radiation power absorbed in walls and electrodes (in a BETA =  
C 1.00 situation, there is NO synchrotron emission, so PABSW would be equal  
C to 0.00 in such a case)

PABSW = FSYNAB \* (1.0-REFSYN) \* PTSYNC ![MW]

C  
C Total power dissipated in electrodes by ohmic heating

POHMIC = IMAXSQ \* RHOCU \* LANODE \* (1.0/(PI\*RANODE\*\*2)+1.0/  
& (PI\*(RCAT\*\*2-RANODE\*\*2))) \* 1.0E-06 \* TIMDIS \* RATREP ![MW]  
print \*, 'POhmic=',POHMIC

C  
C Power removed from magnet

PMAGNT = 0.01 ![MW]

C  
C Total waste power derived from walls and magnet (deuterium coolant flowing  
C through electrodes escapes out into the chamber with the heat of POHMIC  
C in it)

PWASTE = PABSW + PMAGNT + (1.0-REFBRE)\*PTBREM ![MW]

PDELTO = PDELTO - PABSW - POHMIC - PMAGNT

```

C Electric power is % of waste power recovered with 100% efficient turbine and
C 30% efficient generator
C
C     POWELC = 0.30 * PWASTE  ![MW]
C
C Mass flow of H2 at 2000K into the turbine--minimum flow rate to avoid
C overheating of turbine, if ALL of PWASTE is needed to charge the
C capacitor and turbine temperature cannot exceed 2000K
C
C     FLOW = PWASTE * 70.0 / (2000.0-20.0)  ![kg/s]
C
C Power needed beyond POWELC for ECAP; 1.0/0.3 factor is due to 30% efficiency
C
C     PCAPAD = ECAP * RATREP - POWELC  ![MW]
C
C     IF (PCAPAD.GT.0.0) THEN
C         FLOWAD = 0.0  ![kg/s]
C         print *, 'No excess propellant.'
C     ELSEIF (PCAPAD.LT.0.0) THEN
C         FLOW = FLOW + 1.0/0.3 * PCAPAD * 70.0 /
C &         (2000.0-20.0)  ![kg/s]
C         FLOWAD = -1.0 * 1.0/0.3 * PCAPAD * 70.0 /
C &         (2000.0-20.0)  ![kg/s]
C     ENDIF
C
C Deuterium coolant flows through anode and between anode and cathode to cool
C the electrodes between pulses (assume it behaves as the H2 gas and is also
C at 2000K) escapes into the mixing chamber.
C Amount of deuterium needed is calculated assuming electrodes are made of
C copper
C
C     FLOWD = (1.6E+06*POHMIC * 100.0 * 1.0E-04 * 0.009 * 100.0) /
C &     (14.28 * 1000.0 * 1300.0)
C
C Assumed turbine exit temperature = 700K
C
C Gas, having passed through the turbine (at 700K), is injected into the stream
C flowing out of the mixing chamber, which includes the excess flow
C of heated propellant at 2000K and the deuterium coolant flowing out of the
C copper electrodes.
C
C     TAV = ((FLOW-FLOWAD)*700.0 + (FLOWAD+FLOWD)*2000.0) / FLOW  ![K]
C
C Assume gas absorbs all fusion power, mixes uniformly, and exits at uniform
C temperature. Assume gas absorbs heat up to 5000K as H2 with GAMMA = 1.40 .
C Thus, power removed from system is:
C
C     PTDIS = (FLOW+FLOWD) * CPH2 * (TDISON-TAV) + PCAPAD  ![MW]
C     IF (PDELTO-POWIN.LE.0.0) THEN
C         PTDIS = 0.0
C         PRINT *, 'There is no power remaining to heat the plasma'
C         PRINT *, 'gases to 5000K.'

```

```

ELSEIF (PTDIS.GE.(PDELTO-POWIN)) THEN
    PTDIS = PDELTO - POWIN
    TAV = TDISON - (PTDIS - PCAPAD) / ((FLOW+FLOWD)*CPH2)
ENDIF
print *, 'PTDIS(MW)=' ,PTDIS

```

C  
C PLEFT is power left to be absorbed by an assumed completely dissociated and  
C ionized combination of electron and hydrogen ion gases (at 5000K)  
C

```

IF (TAV.GT.5000.0) THEN
    PLEFT = PDELTO - POWIN - PTDIS  ![MW]
ELSE
    PLEFT = 0.0
ENDIF
PRINT *, 'PLEFT(MW)=' ,PLEFT

```

C  
C Power absorbed by the gases  
C

```

IF (TAV.LE.TDISON) THEN
    TSTAG = TAV
ELSE
    TSTAG = TDISON + PLEFT / ((FLOW+FLOWD) * (CPHION*0.999455 +
&      0.000545*CPE))  ![K]
ENDIF

```

```

TTHROT = TSTAG / 1.35  ![K]
VTHROI = SQRT(2.0*CPHION*(TSTAG-TTHROT))  ![m/s]
VTHROE = SQRT(2.0*CPE*(TSTAG-TTHROT))  ![m/s]

```

C  
C Flow exits twice as fast because of nozzle expansion  
C

```

VEXI = 2.0 * VTHROI  ![m/s]
VEXE = 2.0 * VTHROE  ![m/s]

```

C  
C Resultant thrust  
C

```

THRE = 5.45E-04 * (FLOW+FLOWD) * VEXE  ![N]
THRION = 0.999455 * (FLOW+FLOWD) * VEXI  ![N]
THRPRO = THRE + THRION  ![N]
THRTOT = THRPRO  ![N]
FFBURN = (NPIND+NPINH3) / (NDBEG+NH3BEG)

```

```

RETURN
END

```

C=====

C

C MARS MISSION & OTHER SYSTEM CALCULATIONS

C

C=====

SUBROUTINE MARSSYS(STEPS)

```

IMPLICIT REAL*8 (A - Z)

INTEGER STEPS

INCLUDE 'BLOCK.DPF'

100 Metric tons payload mass (1 metric ton = 1000 kg)

MPAYLD = 1.0E+05  ![kg]

Mass of propellant and mass of propellant system and structure

MPROP = (FLOW+FLOWD) * DAYS * 86400.0  ![kg]
MPROST = 0.10 * MPROP  ![kg]

Mass of deuterium and He-3 fuel

MFUEL = 8.703 * RHOIN * VOLINI * RATREP * DAYS * 86400.0  ![kg]
MFUSYS = 0.1 * MFUEL  ![kg]

Convert ECAP to kJ and find capacitor mass (noting CPCAP = 2 kJ/kg)

MCAP = ECAP * 1000.0 / CPCAP  ![kg]

LiH shield thickness

THICK = 0.1 * LOG(0.125*(RRDDN+RRDT) * TIMPIN * RATREP * VOLPIN
&      * DAYS / (4.0*PI*1.157E+12))  ![m]

Density of LiH = 725 kg/m**3

MSHELD = 725.0 * PI * THICK  ![kg]

Mass of magnet

MMAGNT = 67.55  ![kg]

Total mass

MTOTAL = MPAYLD + MCAP + MPROP + MPROST + MFUEL + MSHELD
&      + MFUSYS + MMAGNT  ![kg]

Isp

ISP = THR TOT / (GRAV * (RHOIN*VOLINI*RATREP+FLOW+FLOWD))  ![s]

RETURN
END

```

---

## RESULTS

C  
C=====

```
SUBROUTINE RESULTS(STEPS)

IMPLICIT REAL*8 (A - Z)

INTEGER I,J,K,STEPS

INCLUDE 'BLOCK.DPF'

OPEN (UNIT=2,FILE='DPF.OUT',STATUS='NEW')

WRITE(2,*) 'Initial fill gas density = ',RHOIN
WRITE(2,*)
WRITE(2,*) 'Needed current (in Amps) = ',IMAX,', when'
WRITE(2,*) ' Plasma pinch temperature = ',KTPIN
WRITE(2,*)
WRITE(2,*) 'Initial fractions of D and 3He:'
WRITE(2,*) 'D ',FRACD,'3He',FRACH3
WRITE(2,*)
WRITE(2,*) 'Initial and Final Number Density (in m**-3):'
WRITE(2,*) 'D',NDBEG,NPIND
WRITE(2,*) '3He',NH3BEG,NPINH3
WRITE(2,*) '4He',NH4BEG,NPINH4
WRITE(2,*) 'P',NPBEG,NPINP
WRITE(2,*) 'T',NTBEG,NPINT
WRITE(2,*) 'Fraction of D-3He orig. # densities burnt = ', FFBURN
WRITE(2,*)
WRITE(2,*) 'Rx Rate Parameters for DHE3, DDN, DDP: '
WRITE(2,*) '(in m**3/s)'
WRITE(2,*) SVDH3
WRITE(2,*) SVDDN
WRITE(2,*) SVDDP
WRITE(2,*)
WRITE(2,*) 'Beginning and ending rx rates for DHE3, DDN, DDP:'
WRITE(2,*) '(in m**-3*s**-1)'
WRITE(2,*) RRDH31,RRDH3
WRITE(2,*) RRDDN1,RRDDN
WRITE(2,*) RRDDP1,RRDDP
WRITE(2,*)
WRITE(2,*) 'Charged particle fusion power of DHE3, DDN, DDP:'
WRITE(2,*) '(in MW)'
WRITE(2,*) PFTDH3,PFTDDN,PFTDDP
WRITE(2,*) '-----> Total fusion power (in MW) = ',PFT
WRITE(2,*)
WRITE(2,*) 'Power needed to operate focus (in MW):'
WRITE(2,*) POWIN
WRITE(2,*) 'Power losses for Brem and Sync Radiation (in MW):'
WRITE(2,*) (1.0-REFBRE)*PTBREM,(1.0-REFSYN)*PTSYNC
WRITE(2,*) 'Total radiation power losses (in MW):'
WRITE(2,*) PLOSTO
```

```

WRITE(2,*)

IF (0.3*PDELTO-POWIN-PMAGNT.LT.0.0) THEN
    WRITE(2,*) 'Net DECREASE in power...'
    WRITE(2,*) PDELTO-POWIN-PMAGNT,' [MW]'
ELSEIF (0.3*PDELTO-POWIN-PMAGNT.GT.0.0) THEN
    WRITE(2,*) 'Net INCREASE in power...'
    WRITE(2,*) 0.3*PDELTO-POWIN-PMAGNT,' [MW]'
ELSE
    WRITE(2,*) 'NO NET CHANGE in reactor power.'
ENDIF

WRITE(2,*) 'Total electrical power produced = ',POWELC,' [MW]'
WRITE(2,*)
WRITE(2,*) 'H mass flow rate (kg/s) = ',FLOW+FLOWAD
WRITE(2,*) 'D mass flow rate (kg/s) = ',FLOWD
WRITE(2,*)
WRITE(2,*) 'Plasma stagnation temperature (in K, eV):'
WRITE(2,*) TSTAG,TSTAG/12000.0
WRITE(2,*) 'Ion and electron exit velocities (in m/s):'
WRITE(2,*) VEXI,VEXE
WRITE(2,*) 'Final propellant thrust (in N):'
WRITE(2,*) THRPRO
WRITE(2,*) 'Firing time = ',DAYS,' days'
WRITE(2,*)
WRITE(2,*) '=====The System=====
WRITE(2,*) '          Area                      Mass (kg)
WRITE(2,*) '          ----                      -
WRITE(2,*) 'PAYLOAD                      ',MPAYLD
WRITE(2,*) 'PROPELLANT                    ',MPROP
WRITE(2,*) 'PROPELLANT SYSTEM AND STRUCTURE',MPROST
WRITE(2,*) 'FUEL                          ',MFUEL
WRITE(2,*) 'FUEL SYSTEM                    ',MFUSYS
WRITE(2,*) 'CAPACITOR                     ',MCAPI
WRITE(2,*) 'SHIELD                        ',MSHIELD
WRITE(2,*) 'MAGNET                         ',MMAGNT
WRITE(2,*) '-----
WRITE(2,*) 'TOTAL MASS                      ',MTOTAL
WRITE(2,*) '=====
WRITE(2,*)
WRITE(2,*) 'Total thrust (in Newtons, Lbf):'
WRITE(2,*) THRTOT,THRTOT/4.4482
WRITE(2,*)
WRITE(2,*) 'Thrust to Weight Ratio =',THRTOT / (MTOTAL*GRAV)
WRITE(2,*)
WRITE(2,*) 'Isp (Specific Impulse) = ',ISP,' [s]'
WRITE(2,*)
WRITE(2,*) 'Thrust * Isp                  = ',THRTOT * ISP,' [N-s]'
WRITE(2,*)
WRITE(2,*) '!!!!!!!!!!END!!!!!!!!!!OF!!!!!!!!!!OUTPUT!!!!!!!!!!DATA!!!!!!!!!!'

CLOSE (UNIT=2)

```

RETURN  
END



## APPENDIX B

SYNCHROTRON RADIATION REFLECTION COEFFICIENT  
00  
ANODE RADIUS IN METERS (0.0508)  
0508  
CATHODE RADIUS IN METERS (0.0800)  
1050  
ANODE LENGTH IN METERS (0.382)  
0382  
INITIAL FILL GAS DENSITY IN  $\text{KG/M}^3$  ( $2.2\text{E-4}$ )  
02E-4  
REFLECTED PBR FRACTION DUE TO SEEDING OF PLASMA FUEL  
00  
INITIAL INDUCTANCE  
02E-8  
CAPACITOR BANK SPECIFIC ENERGY IN KJ/KG  
00  
FLOWPLOW EFFICIENCY FACTOR  
070  
PINCH EFFICIENCY FACTOR  
025  
PINCH LENGTH IN METERS  
00254  
PINCH RADIUS IN METERS  
05E-3  
FRACTION OF DEUTERIUM IN INITIAL FILL GAS  
055  
FRACTION OF HELIUM-3 IN INITIAL FILL GAS  
045  
WANTED OPERATING TEMPERATURE OF PINCH (keV)  
00.0  
PLASMA FOCUS FIRING RATE IN  $\text{S}^{*-1}$   
00.0  
DURATION OF STABLE PINCH IN SECONDS  
000E-4  
NUMBER OF ITERATIONS ( $\geq 1000$  for accurate modeling)  
000  
TOTAL FIRING TIME (for entire mission to Mars w/ $\Delta v$  of 30 km/s) IN DAYS  
00.0